Forward gradients

Gradients without Backpropagation

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SCALING FORWARD GRADIENT WITH LOCAL LOSSES

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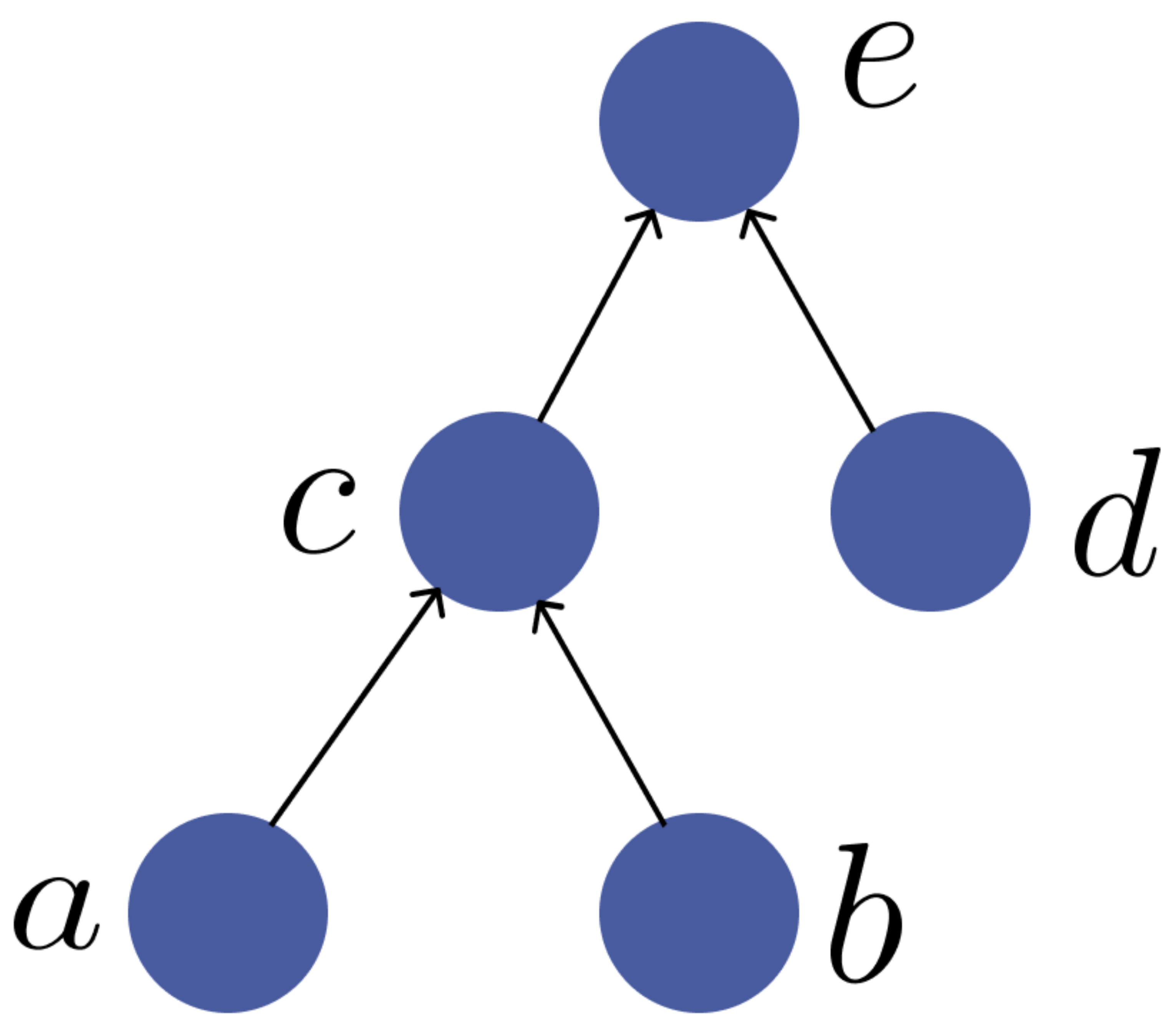
Seth, Tea Talk 07/11/23

Forward or backward?

$$\frac{\partial \mathbf{f}_\theta}{\partial \theta_{l,i}} = \frac{\partial \mathbf{f}_\theta}{\partial \mathbf{x}_L} \frac{\partial \mathbf{x}_L}{\partial \mathbf{x}_{L-1}} \cdots \frac{\partial \mathbf{x}_{l+1}}{\partial \mathbf{x}_l} \frac{\partial \mathbf{x}_l}{\partial \theta_{l,i}}$$

- **Backprop**: compute function $\mathbf{f}_\theta(\mathbf{x}_0)$ and right-multiply, working backwards through the network
- **Forwardprop:** multiply Jacobians, $\left\{\frac{\partial \mathbf{x}_{l+1}}{\partial \mathbf{x}_l}\right\}$, from R to L in same forward pass as evaluating $\mathbf{f}_\theta(\mathbf{x}_0)$

Why backprop?

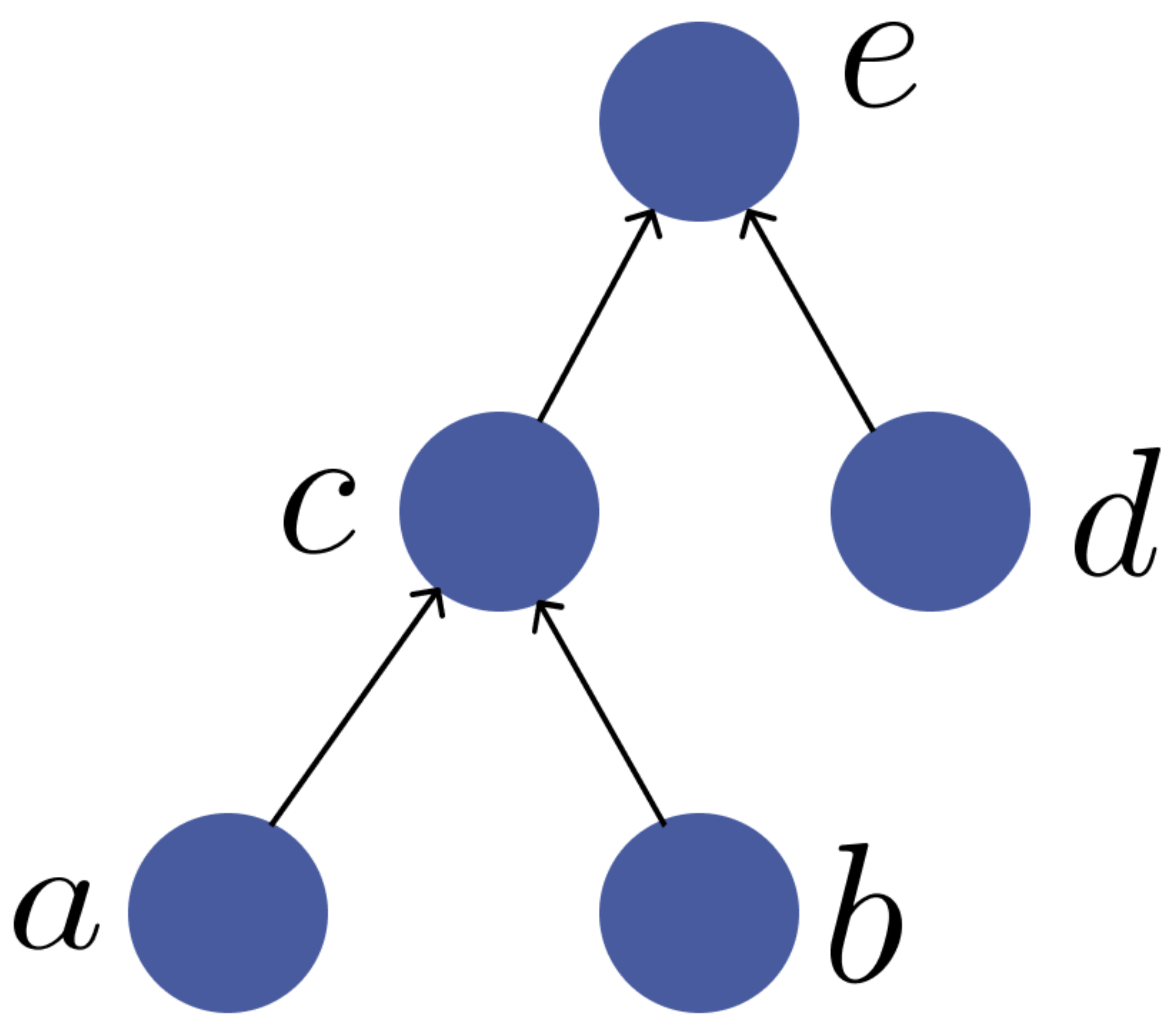


- To compute e.g. $\partial e / \partial a$ with forwardprop
  - First compute $\partial c / \partial a$ and $c(a, b)$
  - Then compute $\partial e / \partial c$ and "push-forward"

$$\frac{\partial e}{\partial a} = \frac{\partial e}{\partial c} \cdot \frac{\partial c}{\partial a}$$

- Memory efficient: don't need to store intermediates $c(a, b)$
- But if we now want e.g. $\partial e / \partial b$, we have to repeat this procedure from scratch

Why backprop?



- With backprop
  - Evaluate e once,
  - Use this (+ intermediates) to compute gradients w.r.t all variables

Why backprop?

- In general, to compute gradient of function $\mathrm{f}_\theta : \mathbb{R}^N \to \mathbb{R}^M$ w.r.t $\theta \in \mathbb{R}^D$

	Forward prop	**Backprop**
Time	$O(FD)$	$O(FM)$
Memory	$O(1)$	$O(L)$

where F is complexity of evaluating $\mathrm{f}_\theta(\cdot)$ and L is depth

- In ML, usually $M << D$ and so backprop is more time-efficient

Directional derivatives

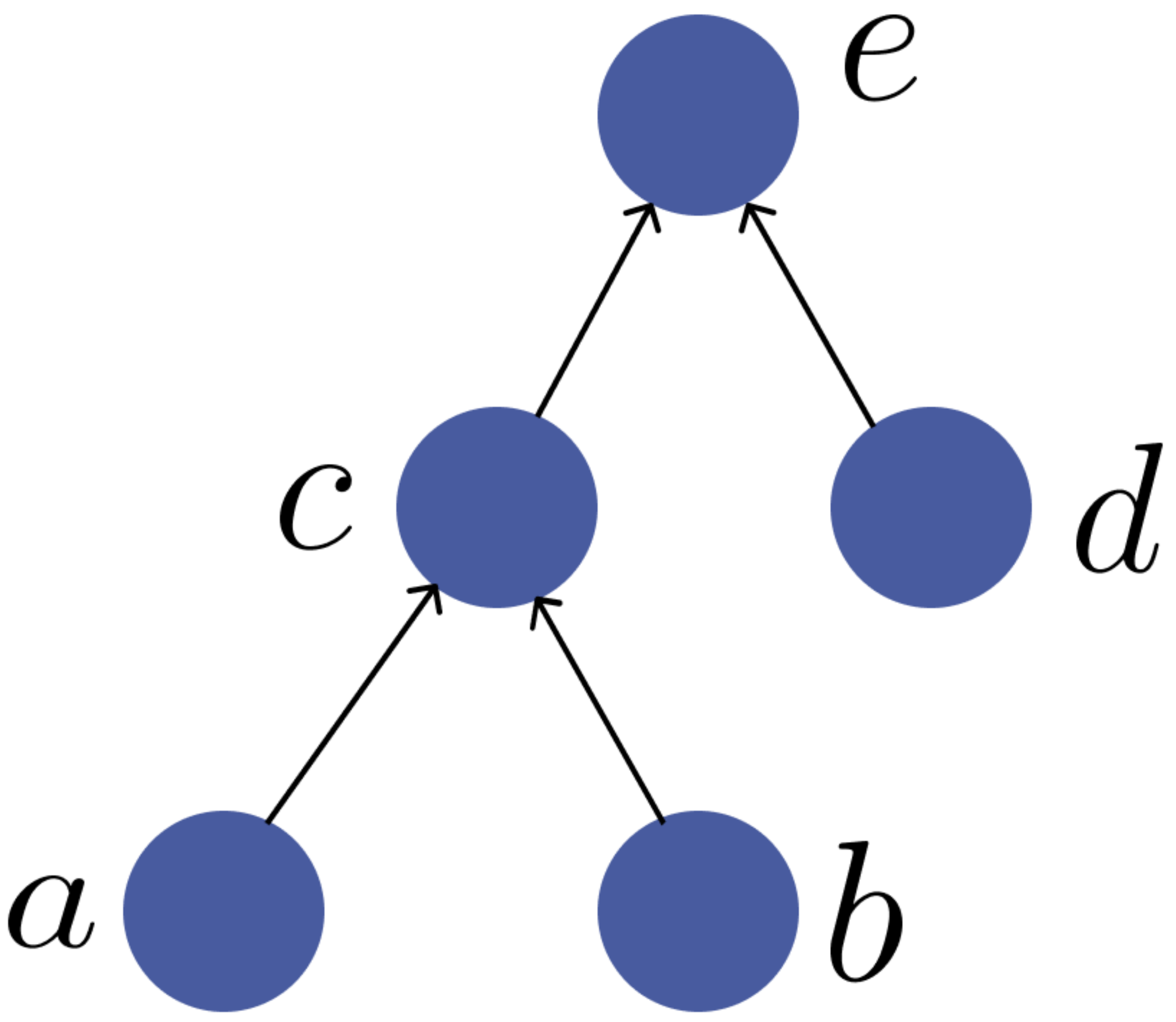
Gradients without Backpropagation

Atılım Güneş Baydin [1] Barak A. Pearlmutter [2] Don Syme [3] Frank Wood [4] Philip Torr [5]

- To make forward prop efficient, Baydin et al use it for the directional derivative $\nabla \mathbf{f}_\theta \cdot \mathbf{v}$ along a random vector $\mathbf{v} \sim p(\mathbf{v})$
- If $\{v_i\}$ are iid, zero-mean and unit-variance; the expression $(\nabla \mathbf{f}_\theta \cdot \mathbf{v}) \cdot \mathbf{v}$ is an unbiased estimator for $\nabla \mathbf{f}_\theta$ (proof to follow)
- Note that $\nabla \mathbf{f}_\theta$ is never actually computed

Baydin, Pearlmutter, Syme, Wood, Torr. Gradients without Backpropagation, 2022

Directional derivatives



- To estimate $\nabla e = \left[\frac{\partial e}{\partial a}, \frac{\partial e}{\partial b}, \frac{\partial e}{\partial d}\right]^T$

$$
\begin{aligned}
\nabla e &= \left[\frac{\partial e}{\partial a}, \frac{\partial e}{\partial b}, \frac{\partial e}{\partial d}\right]^T \\
&\approx (\nabla e \cdot \mathbf{v}) \cdot \mathbf{v}, \quad \text{where } \mathbf{v} = [v_a, v_b, v_d]^T \sim p(\mathbf{v}) \\
&\approx \left(\frac{\partial e}{\partial a} \cdot v_a + \frac{\partial e}{\partial b} \cdot v_b + \frac{\partial e}{\partial d} \cdot v_d\right) \cdot \mathbf{v} \\
&\approx \left(\frac{\partial e}{\partial c}\left(\frac{\partial c}{\partial a} \cdot v_a + \frac{\partial c}{\partial b} \cdot v_b\right) + \frac{\partial e}{\partial d} \cdot v_d\right) \cdot \mathbf{v}
\end{aligned}
$$

Unbiasedness proof

- The inner product is

$$\nabla \mathbf{f}_\theta \cdot \mathbf{v} = \sum_k \frac{\partial f}{\partial \theta_k} v_k = \frac{\partial f}{\partial \theta_i} v_i + \sum_{j \neq i} \frac{\partial f}{\partial \theta_j} v_j$$

- Therefore, the *i*th element of $\mathbb{E}_{\mathbf{v}}\left[(\nabla \mathbf{f}_\theta \cdot \mathbf{v}) \cdot \mathbf{v}\right]$ is

$$\frac{\partial f}{\partial \theta_i} \mathbb{E}\left[v_i^2\right] + \sum_{j \neq i} \frac{\partial f}{\partial \theta_j} \mathbb{E}\left[v_i v_j\right]$$

- $p(\mathbf{v})$ is chosen s.t. v_i are mean zero, variance one, and i.i.d
    - **Mean zero + variance one** — first expectation is 1
    - **I.i.d + mean zero** — second expectation is 0
- Thus *i*th element of $\mathbb{E}_{\mathbf{v}}\left[(\nabla \mathbf{f}_\theta \cdot \mathbf{v}) \cdot \mathbf{v}\right]$ is *i*th element of the true gradient

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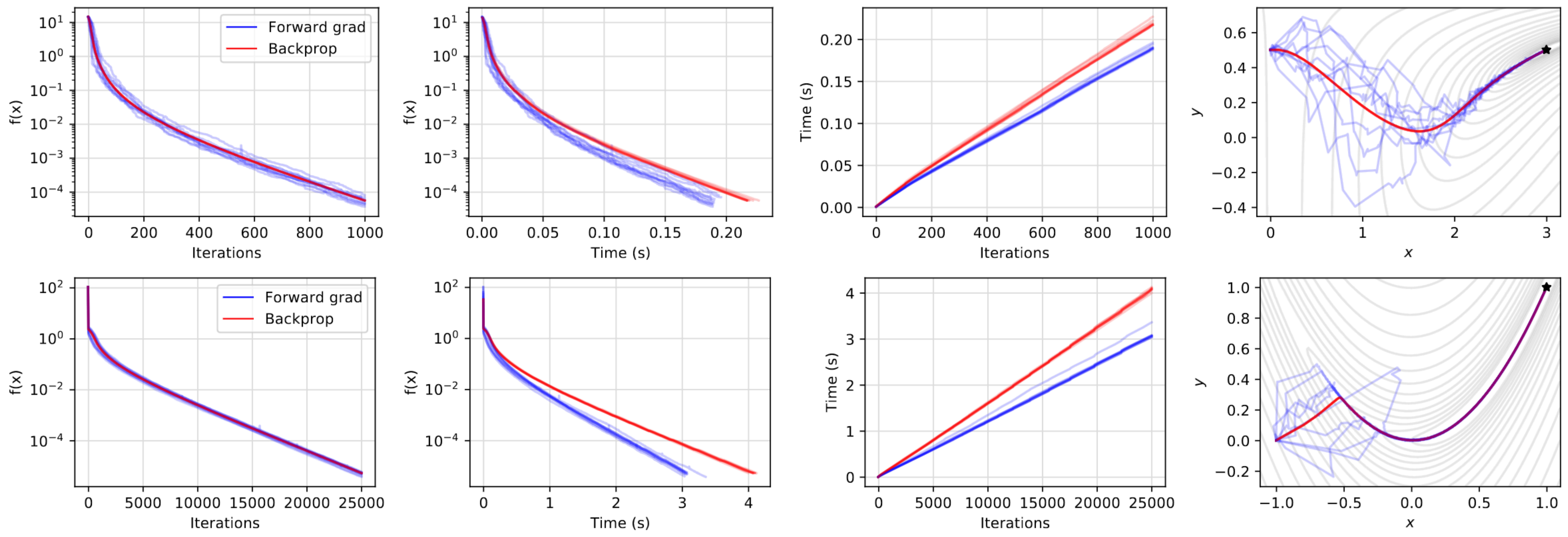
Results

Toy optimisation problems



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Results

MNIST, MLP

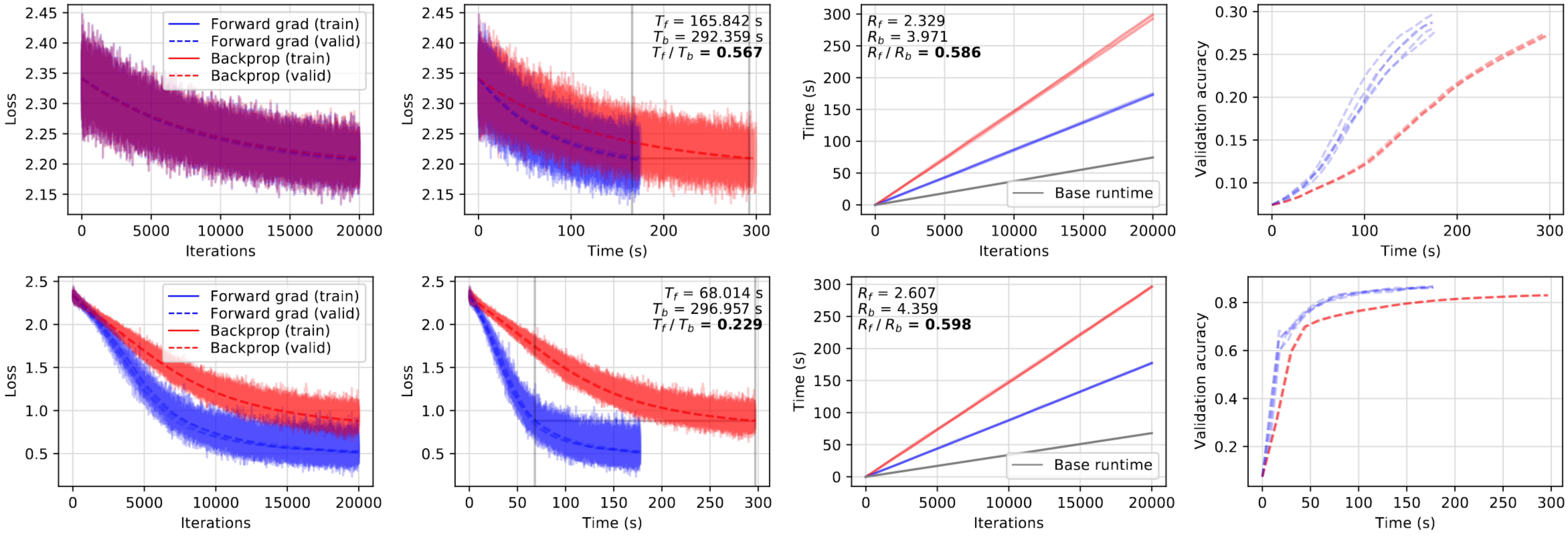

Figure 4. Comparison of forward gradient and backpropagation for the multi-layer NN, showing two learning rates. Top row: learning rate 2×10^{-5}. Bottom row: learning rate 2×10^{-4}. Showing five independent runs per experiment.

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Results

MNIST, CNN

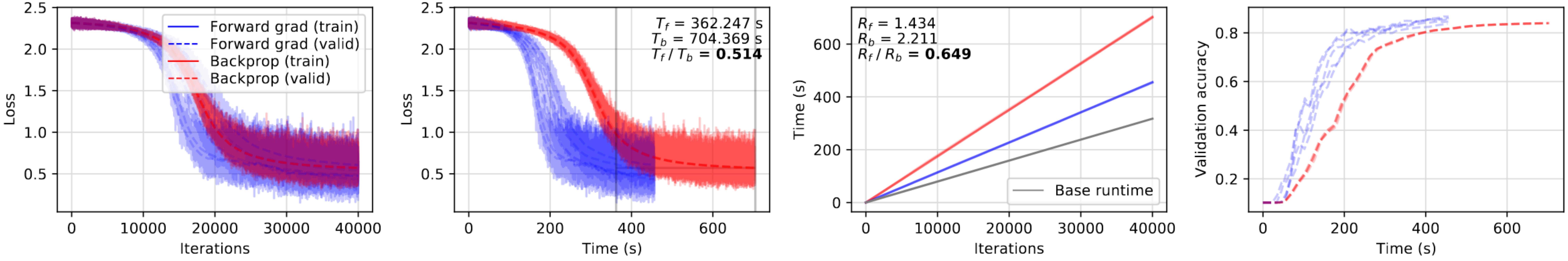

Figure 5. Comparison of forward gradient and backpropagation for the CNN. Learning rate 2×10^{-4}. Showing five independent runs.

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Results

Runtime and memory

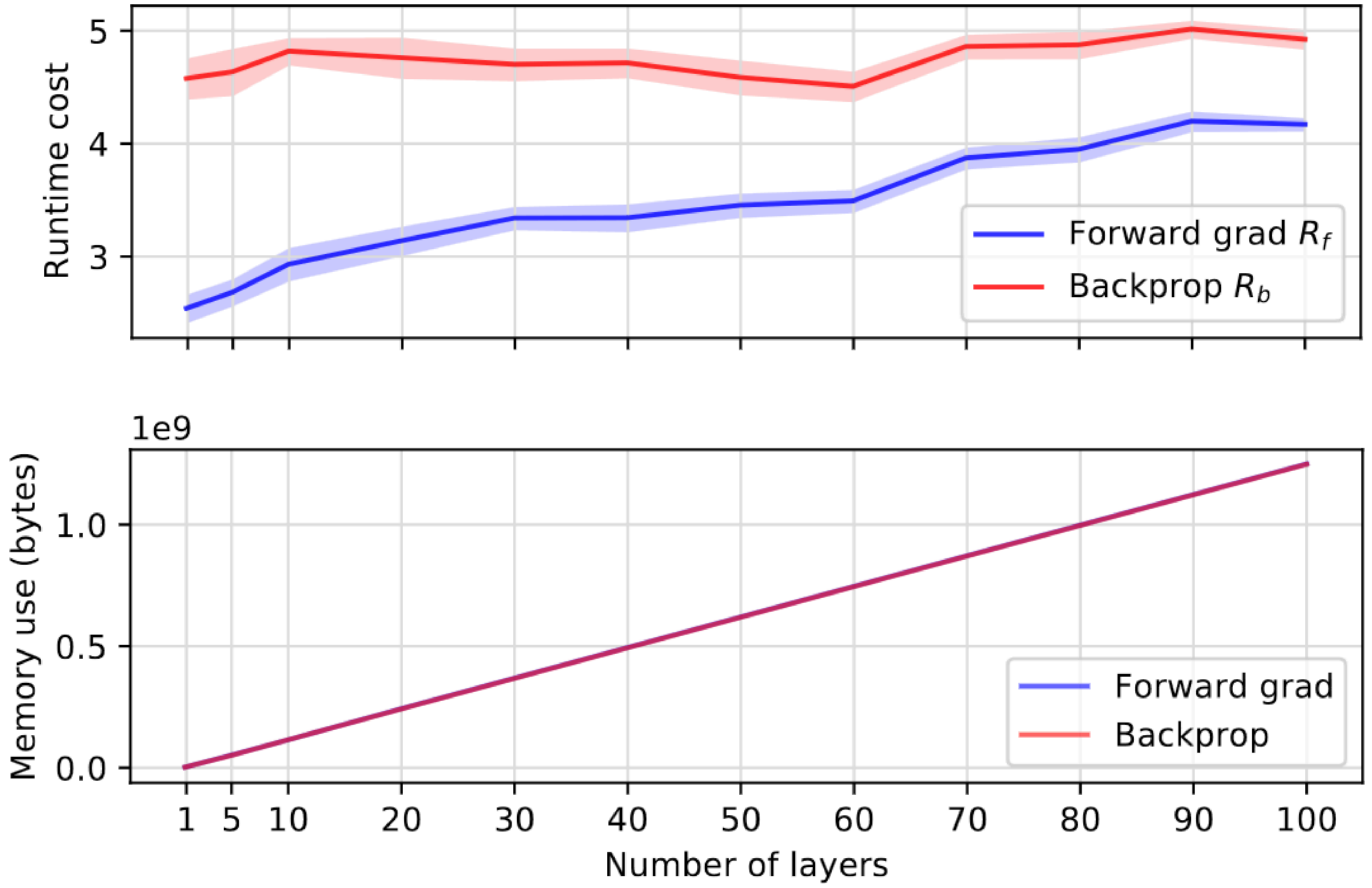

Figure 6. Comparison of how the runtime cost and memory usage of forward gradients and backpropagation scale as a function NN depth for the MLP architecture where each layer is of size 1024. Showing mean and standard deviation over ten independent runs.

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Scaling Forward Propagation

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- Reduces variance of forward gradients:
  - Apply perturbation vector $\mathbf{v}$ to activation gradients instead of weights
  - Add local losses on blocks, spatial patches and groups of channels
- "LocalMixer" architecture inspired by MLPMixer
- Experiments scaled to ImageNet

Results
Supervised learning

Dataset Network Metric	MNIST S/1/1 Test / Train Err. (%)	MNIST M/1/16 Test / Train Err. (%)	CIFAR-10 M/8/16 Test / Train Err. (%)	ImageNet L/32/64 Test / Train Err. (%)
BP	2.66 / 0.00	2.41 / 0.00	33.62 / 0.00	36.82 / 14.69
L-BP	2.38 / 0.00	2.16 / 0.00	30.75 / 0.00	42.38 / 22.80
LG-BP	2.43 / 0.00	2.81 / 0.00	33.84 / 0.05	54.37 / 39.66
BP-free algorithms				
FA	**2.82 / 0.00**	2.90 / **0.00**	39.94 / 28.44	94.55 / 94.13
L-FA	3.21 / **0.00**	2.90 / **0.00**	39.74 / 28.98	87.20 / 85.69
LG-FA	3.11 / **0.00**	**2.50 / 0.00**	39.73 / 32.32	85.45 / 82.83
DFA	3.31 / **0.00**	3.17 / **0.00**	38.80 / 33.69	91.17 / 90.28
FG-W	9.25 / 8.93	8.56 / 8.64	55.95 / 54.28	97.71 / 97.58
FG-A	3.24 / 1.53	3.76 / 1.75	59.72 / 41.29	98.83 / 98.80
LG-FG-W	9.25 / 8.93	5.66 / 4.59	52.70 / 51.71	97.39 / 97.29
LG-FG-A	3.24 / 1.53	2.55 / **0.00**	**30.68 / 19.39**	**58.37 / 44.86**

Table 3: Supervised learning for image classification

BP = Backprop

L = Local losses

LG = Greedy local losses

FA = Feedback alignment (random, fixed backward weights)

DFA = Direct feedback alignment

FG = Forward gradients

W = Weight-perturbed (Baydin et al)

A = Activation-perturbed

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Results

Self-supervised learning

Dataset Network Metric	CIFAR-10 M/8/16 Test / Train Err. (%)	CIFAR-10 L/8/64 Test / Train Err. (%)	ImageNet L/32/64 Test / Train Err. (%)
BP	24.11 / 21.08	17.53 / 13.35	55.66 / 49.79
L-BP	24.69 / 21.80	19.13 / 13.60	59.11 / 52.50
LG-BP	29.63 / 25.60	23.62 / 16.80	68.36 / 62.53
BP-free algorithms			
FA	45.87 / 44.06	67.93 / 65.32	82.86 / 80.21
L-FA	37.73 / 36.13	31.05 / 26.97	83.18 / 79.80
LG-FA	36.72 / 34.06	30.49 / 25.56	82.57 / 79.53
DFA	46.09 / 42.76	39.26 / 37.17	93.51 / 92.51
FG-W	53.37 / 51.56	50.45 / 45.64	91.94 / 89.69
FG-A	54.59 / 52.96	56.63 / 56.09	97.83 / 97.79
LG-FG-W	52.66 / 50.23	52.27 / 48.67	91.36 / 88.81
LG-FG-A	**32.88 / 29.73**	**26.81 / 23.90**	**73.24 / 66.89**

Table 4: Self-supervised contrastive learning with linear readout

BP = Backprop

L = Local losses

LG = Greedy local losses

FA = Feedback alignment (random, fixed backward weights)

DFA = Direct feedback alignment

FG = Forward gradients

W = Weight-perturbed (Baydin et al)

A = Activation-perturbed

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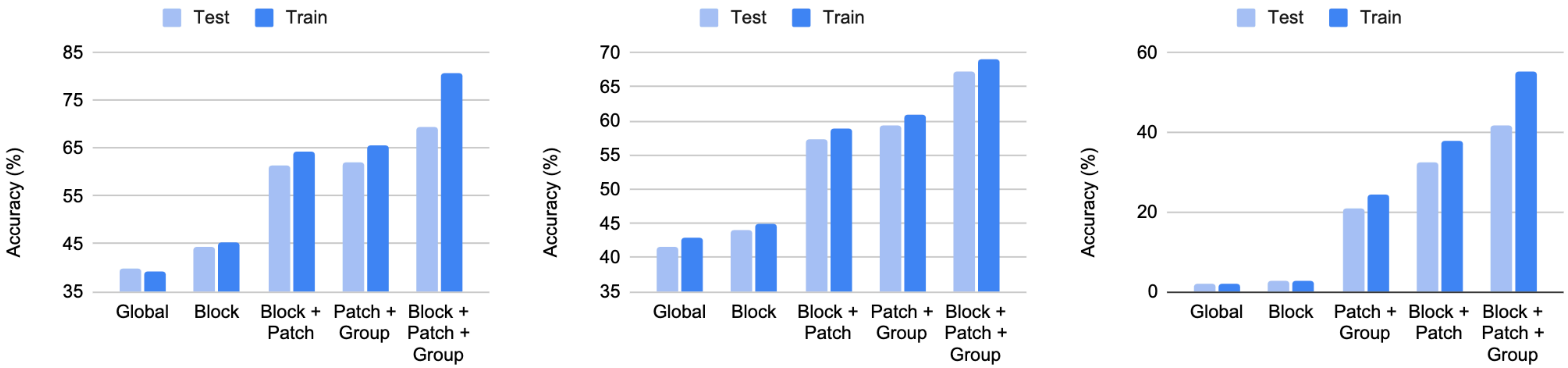
Results
Local losses



(a) CIFAR-10 Supervised M/8 (b) CIFAR-10 Contrastive M/8 (c) ImageNet Supervised L/32

Ren, Kornblith, Liao, Hinton. Scaling Forward Gradients with Local Losses. 2023

Conclusion

- Forward propagation has some compelling properties
  - Reduced memory relative to backprop, particularly for deeper networks
  - Gradient + function eval in a single forward pass
- Limitations for case with few outputs and many inputs addressed by using random directional derivatives (Baydin et al., 2022)
- Scaled to large models and datasets by Ren et al., 2023